West Windsor-Plainsboro Regional School District
Advanced Topics in Physics Honors Curriculum
Grade(s): 11-12

The Mission of the West Windsor-Plainsboro Science Department

Our mission is to cultivate science learners who have the foundational knowledge to make ethical, scientifically literate decisions and the ability to apply scientific practices in order to contribute to the needs of society and a changing world.

- **Vision**

  We envision a K-12 science experience that supports and challenges every student in their science learning journey. We will:
  - *Capitalize on diversity by reaching and exciting students at all levels and interests by differentiating learning within classrooms and by offering a robust program of studies.*
  - *Emphasize authentic science and engineering practices and leverage the interdisciplinary nature of science with arts, technology, math, reading, and writing.*
  - *Integrate scientific knowledge and 21st century competencies to prepare students to make informed decisions and take action to address real world problems.*

Unit Title: Unit 1 - Static Equilibrium & Torque	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11-12	
Summary and Rationale	
In this unit, students will study torque and rotational equilibrium in preparation for the study of rotational kinematics, dynamics, rotational kinetic energy, and angular momentum. Although these topics introduce more complexity, the concepts and analysis from the first year of physics set the foundation for this unit. Throughout this unit and the next, students will have opportunities to make connections to the concepts of translational motion, dynamics, energy, and momentum from the first year of physics to make meaning of these concepts as a whole, rather than as distinct and separate units (e.g., static equilibrium). While some students may easily make the analogy between translational and rotational kinematics and dynamics, some may experience, or re-experience, the conceptual difficulties found in translational motion.	
Recommended Pacing	
11 days	
NGSS Standards/Performance Expectations	
HS-PS2-1	Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. Clarification Statement: Examples of data could include tables or graphs of position or velocity as a function of time for objects subject to a net unbalanced force, such as a falling object, an object rolling down a ramp, or a moving object being pulled by a constant force.
HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.
Interdisciplinary Connections	
Integration of Technology	
Instructional Focus	
Unit Enduring Understandings	
● A body is in equilibrium when it remains either in uniform motion (both translational and rotational) or at rest. When a body in a selected inertial frame of reference neither rotates nor moves in translational motion, we say the body is in static equilibrium in this frame of reference. ● Conditions for equilibrium require that the sum of all external forces acting on the body is zero (first condition of equilibrium), and the sum of all external torques from external forces is zero (second condition of equilibrium). These two conditions must be simultaneously satisfied in equilibrium. If one of them is not satisfied, the body is not in equilibrium. ● The free-body diagram for a body is a useful tool that allows us to count correctly all contributions from all external forces and torques acting on the body. Free-body diagrams for the equilibrium of an	

extended rigid body must indicate a pivot point and lever arms of acting forces with respect to the pivot.

- A variety of engineering problems can be solved by applying equilibrium conditions for rigid bodies.
- In applications, identify all forces that act on a rigid body and note their lever arms in rotation about a chosen rotation axis. Construct a free-body diagram for the body. Net external forces and torques can be clearly identified from a correctly constructed free-body diagram. In this way, you can set up the first equilibrium condition for forces and the second equilibrium condition for torques.
- In setting up equilibrium conditions, we are free to adopt any inertial frame of reference and any position of the pivot point. All choices lead to one answer. However, some choices can make the process of finding the solution unduly complicated. We reach the same answer no matter what choices we make. The only way to master this skill is to practice.

Learning Objectives

Students will be able to:

- Calculate the magnitude and direction of the torque associated with a given force acting on a rigid body system.
- Calculate the torque acting on a rigid body due to the gravitational force.
- Describe the two conditions of equilibrium for an extended rigid body.
- Calculate unknown magnitudes and directions of forces acting on an extended rigid body that is in a state of translational and rotational equilibrium.
- Calculate the center of mass of a system of point masses or a system of regular symmetrical objects.
- Calculate the center of mass of a thin rod of nonuniform density using integration.

Sample Performance Tasks

- Make observations or collect data from representations of laboratory setups or results. (HS-PS2-5)
- Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units. (HS-PS2-1) (HS-PS2-6)
- Make a claim or predict the results of an experiment. (HS-PS2-5)
- Determine the relationship between variables within an equation when an existing variable changes.(HS-PS2-2),(HS-PS2-4)
- Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)
- Describe the effects of modifying conditions or features of a representation of a physical situation. (HS-PS2-2),(HS-PS2-4)
- Sketch a graph that shows a functional relationship between two quantities. (HS-PS2-2),(HS-PS2-4)
- Select relevant features of a graph to describe a physical situation or solve problems. (HS-PS2-5)
- Determine or estimate the change in a quantity using a mathematical relationship. (HS-PS2-2),(HS-PS2-4)
- Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. (HS-PS2-1)
- Assess the reasonableness of results or solutions. (HS-ETS1-3)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- The definition of torque is:

$$\tau = \square \times \square$$

- Torque is a vector product (or cross-product), and it has a direction that can be determined by the vector product or by applying the appropriate right-hand rule.

- The idea of the “moment-arm” is useful when computing torque. The moment arm is the perpendicular distance between the pivot point and the line of action of the point of application of the force. The magnitude of the torque vector is equivalent to the product of the moment arm and the force.
- The two conditions of equilibrium are:
  - $\Sigma\square = 0$
  - $\Sigma\tau = 0$
  - Both conditions must be satisfied for an extended rigid body to be in equilibrium.
- A symmetrical, regular solid of uniform mass density has a center of mass at its geometric center. a.
  - For a nonuniform solid that can be considered as a collection of regular masses or for a system of masses: *Equation*
  - The calculus definition is: *Equation*
- If there is no net force acting on an object or a system, the center of mass does not accelerate; therefore, the velocity of the center of mass remains unchanged.
  - A system of multiple objects can be represented as one single mass with a position represented by the center of mass.
  - The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: <u>Fundamentals of Physics 10th Edition</u>, Halliday & Resnick, ISBN 978-1-118-23072-5 (Extended Edition)

Suggested Resources:

Unit Title: Unit 2 Rotation	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11-12	
Summary and Rationale	
In this unit, students will study rotational kinematics, dynamics, rotational kinetic energy, and angular momentum of rigid bodies about different axes of rotation. Topics include center of mass, rolling objects, vector nature of rotational bodies, moments of inertia, tipping, and parallel axis theorem. Throughout this unit, students will continue to have opportunities to make connections to the concepts of translational motion, dynamics, energy, and momentum from the first year of physics to make meaning of these concepts as a whole, rather than as distinct and separate units (e.g., static equilibrium). While some students may easily make the analogy between translational and rotational kinematics and dynamics, some may experience, or re-experience, the conceptual difficulties found in translational motion. Students' knowledge of angular momentum and its conservation is revisited and built upon in Unit 7 with satellites in orbit.	
Recommended Pacing	
21 days	
NGSS Standards/Performance Expectations	
HS-PS2-1	Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. Clarification Statement: Examples of data could include tables or graphs of position or velocity as a function of time for objects subject to a net unbalanced force, such as a falling object, an object rolling down a ramp, or a moving object being pulled by a constant force.
HS-PS2-2	Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system. [Clarification Statement: Emphasis is on the quantitative conservation of momentum in interactions and the qualitative meaning of this principle.] [Assessment Boundary: Assessment is limited to systems of two macroscopic bodies moving in one dimension.]
HS-PS3-1	Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known. [Clarification Statement: Emphasis is on explaining the meaning of mathematical expressions used in the model.] [Assessment Boundary: Assessment is limited to basic algebraic expressions or computations; to systems of two or three components; and to thermal energy, kinetic energy, and/or the energies in gravitational, magnetic, or electric fields.]
HS-PS3-2	Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as a combination of energy associated with the motions of particles (objects) and energy associated with the relative position of particles (objects). [Clarification Statement: Examples of phenomena at the macroscopic scale could include the conversion of kinetic energy to thermal energy, the energy stored due to position of

	an object above the earth, and the energy stored between two electrically-charged plates. Examples of models could include diagrams, drawings, descriptions, and computer simulations.]
HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.

Interdisciplinary Connections

Integration of Technology

Instructional Focus

Unit Enduring Understandings

- Rotational Motion and Angular Displacement When a rigid body rotates about fixed axis, angular displacement is the angle swept out by an imaginary "radius" of the body Counterclockwise is positive, clockwise is negative Radian is SI unit, defined as arc length divided by radius
- Angular Velocity and Angular Acceleration Average angular velocity is angular displacement divided by time Average angular acceleration is the change in angular velocity divided by time
- Rolling Motion No slipping at the point where object touches the surface As a result, linear equals tangential velocity of the edge point Similarly, tangential acceleration and linear acceleration are equal
- Vector Nature of Angular Variables Right Hand Rule: Grasp axis of rotation with right hand, so fingers circle in the same sense as the rotation. Your thumb points in the direction of the angular velocity, or angular acceleration, variable.
- In the absence of external torques, a system’s total angular momentum is conserved. This is the rotational counterpart to linear momentum being conserved when the external force on a system is zero.
- For a rigid body that changes its angular momentum in the absence of a net external torque, conservation of angular momentum gives $I_i\omega_i = I_f\omega_f$. This equation says that the angular velocity is inversely proportional to the moment of inertia. Thus, if the moment of inertia decreases, the angular velocity must increase to conserve angular momentum.
- Systems containing both point particles and rigid bodies can be analyzed using conservation of angular momentum. The angular momentum of all bodies in the system must be taken about a common axis.

Learning Objectives

Students will be able to:

- Explain the differences in the moments of inertia between different objects such as rings, discs, spheres, or other regular shapes by applying the general definition of moment of inertia (rotational inertia) of a rigid body.
- Calculate by what factor an object’s rotational inertia will change when a dimension of the object is changed by some factor.
- Calculate the moment of inertia of point masses that are located in a plane about an axis perpendicular to the plane.
- Derive the moment of inertia, using calculus, of a thin rod of uniform density about an arbitrary axis perpendicular to the rod.
- Derive the moment of inertia, using calculus, of a thin rod of nonuniform density about an arbitrary axis perpendicular to the rod.

- Derive the moments of inertia for a thin cylindrical shell or disc about its axis or an object that can be considered to be made up of coaxial shells.
- Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.

Sample Performance Tasks

- Make observations or collect data from representations of laboratory setups or results. (HS-PS2-5)
- Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units. (HS-PS2-1) (HS-PS2-6)
- Make a claim or predict the results of an experiment. (HS-PS2-5) (HS-PS3-4)
- Determine the relationship between variables within an equation when an existing variable changes.(HS-PS2-2),(HS-PS2-4)
- Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)
- Describe the effects of modifying conditions or features of a representation of a physical situation. (HS-PS2-2),(HS-PS2-4)
- Sketch a graph that shows a functional relationship between two quantities. (HS-PS2-2),(HS-PS2-4)
- Select relevant features of a graph to describe a physical situation or solve problems. (HS-PS2-5)
- Determine or estimate the change in a quantity using a mathematical relationship. (HS-PS2-2),(HS-PS2-4)
- Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. (HS-PS2-1)
- Assess the reasonableness of results or solutions. (HS-PS3-3) (HS-ETS1-3)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- The general definition of moment of inertia is:

$$I = \sum m_i r_i^2$$

- The calculus definition of moment of inertia is:

$$I = \int r^2 dm$$

  - The differential *dm* must be determined from the linear mass density of the rod or object.
- The parallel axis theorem is a simple powerful theorem that allows the moments of inertia to be computed for an object through any axis that is parallel to an axis through its center of mass.

$$I' = I_{cm} + Md^2$$

- There are angular kinematic relationships for objects experiencing a uniform angular acceleration. These are the relationships:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

- Other relationships can be derived from the above two relationships.
  - The appropriate unit for angular position is radians.
  - The general calculus kinematic linear relationships have analogous representations in rotational motion such as:

$$\omega = \frac{d\theta}{dt}$$

- For objects that are rolling without slipping on surface, the angular motion is related to the linear translational motion by the following relationships:

$$\square = \square\square$$
$$\square = \square\square$$
$$\Delta\square = \square\square\square$$

- The rotational analog to Newton's second law is:

$$\alpha = \frac{\square\square}{\square}$$

    - In the appropriate cases, both laws (Newton's second law and the analogous rotational law) can be applied to a dynamic system and the two laws are independent from each other.
- All real forces acting on an extended rigid body can be represented by a rigid body diagram. The point of application of each force can be indicated in the diagram.
    - The rigid body diagram is helpful in applying the rotational Newton's second law to a rotating body.
- A complete analysis of a dynamic system that is rolling without slipping can be performed by applying both of Newton's second laws properly to the system.
    - The rotational characteristics may be related to the linear motion characteristics (i.e., $\square = \square\square$)
    - If the rigid body undergoing motion has a rotational component of motion and an independent translational motion (i.e., the object is slipping), then the rolling condition relationships do not hold.

$$\square \neq \square\square$$

- The definition of rotational kinetic energy is:

$$\square_{\square} = \frac{1}{2}\square\square^{2}$$

    - Total kinetic energy of a rolling body or a body with both forms of motion is the sum of each kinetic energy term.
    - The definition of work also has an analogous form in rotational dynamics:

$$\square = \int \square\square\square$$

- If a rigid body is defined as "rolling," this implies (in the ideal case) that the frictional force does not work on the rolling object. The consequence of this property is that in some special cases (such as a sphere rolling down an inclined surface), the conservation of mechanical energy can be applied to the system.
- The definition of angular momentum of a rotating rigid body is:

$$\square = \square\square$$

    - Angular impulse is equivalent to the change in angular momentum. The definition of this relationship is:

$$\int \tau\square\square = \square\square$$

    - The differential definition is:

$$\tau = \frac{\square\square}{\square\square}$$

- The angular momentum of a linearly translating particle can be defined about some arbitrary point of reference or origin. The definition is:

$$\square = \square \times \square$$

    - The direction of this particle's angular momentum is determined by the vector product (cross-product).

● In the absence of external torques acting on a rotating body or system, the total angular momentum of the system is constant.
● The conservation of angular momentum can be applied to many types of physical situations. In all cases, it must be determined that there is no net external torque on the system.
  ○ In the case of collisions (such as two discs colliding with each other), the torques applied to each disc are "internal" if the system is considered to be the two discs.
  ○ In the case of a particle colliding with a rod or physical pendulum, the system is considered to be the particle and the rod together.

Competencies for 21st Century Learners

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	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Fundamentals of Physics 10th Edition, Halliday & Resnick, ISBN 978-1-118-23072-5 (Extended Edition)

Suggested Resources:

https://www.nextgenscience.org/sites/default/files/HS%20PS%20DCI%20combined%206.13.13.pdf

https://apcentral.collegeboard.org/courses/ap-physics-c-mechanics?course=ap-physics-c-mechanics

https://apcentral.collegeboard.org/courses/ap-physics-c-electricity-and-magnetism?course=ap-physics-c-electricity-and-magnetism

Unit Title: Unit 3 Electrostatics and Fields	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11-12	
Summary and Rationale	
In this unit, students will continue the study of electric force and fields from the first year of physics with the incorporation of integration techniques to calculate physical quantities dealing with electrostatics and fields. Mathematical techniques will be upgraded in order to analyse and describe how distributions of charges, rather than just ideal point charges, interact with other phenomena. This unit will also dive deeper into the realm of the electric potential, and start to explain the reasoning behind how electrical circuits work in the first place, which will be important for our following unit dealing with circuits.	
Recommended Pacing	
20 days	
NGSS Standards/Performance Expectations	
HS-PS2-4	Use the mathematical representation of Coulomb's Law to describe and predict the electrostatic forces between objects. (Clarification Statement: Emphasis is on both quantitative and conceptual descriptions of electric fields.)
HS-PS2-5	Plan and conduct an investigation to provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.
HS-PS3-5	Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction. (Clarification Statement: Examples of models could include drawings, diagrams, and texts, such as drawings of what happens when two charges of opposite polarity are near each other.)
HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
Interdisciplinary Connections	
Integration of Technology	
Instructional Focus	
Unit Enduring Understandings	

- Objects with an electric charge will interact with each other by exerting forces on each other.
- Coulomb's law provides the mathematical models to describe and predict the effects of electrostatic forces between distant objects. Forces at a distance are explained by fields permeating space that can transfer energy through space.
- Objects with an electric charge will create an electric field.
- Attraction and repulsion between electric charges at the atomic scale explain the structure, properties, and transformations of matter, as well as the contact forces between material objects..
- Phenomena at the macroscopic scale are better understood at the microscopic scale, at which all of the different manifestations of energy can be modeled as either motions of particles or energy stored in fields (which mediate interactions between particles).
- Electric and magnetic force fields contain energy and can transmit energy across space from one object to another. When two objects interacting through a force field change relative position, the

energy stored in the force field is changed. Each force between the two interacting objects acts in the direction such that motion in that direction would reduce the energy in the force field between the objects. However, prior motion and other forces also affect the actual direction of motion.
- The total energy of a system composed of a collection of point charges can transfer from one form to another without changing the total amount of energy in the system.
- There are laws that use symmetry and calculus to derive mathematical relationships that can be applied to physical systems containing electrostatic charge.
- There are laws that use calculus and symmetry to derive mathematical relationships that can be applied to electrostatic-charge distributions.

Learning Objectives

Students will be able to:
- Describe behavior of charges or system of charged objects interacting with each other.
- Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
- Use integration techniques to calculate a potential difference between two points on a line, given the electric field as a function of position on that line.
- State and apply the general definition of electric flux.
- Calculate the electric flux through an arbitrary area or through a geometric shape (e.g., cylinder, sphere).
- Calculate the flux through a rectangular area when the electric field is perpendicular to the rectangle and is a function of one position coordinate only.
- Qualitatively apply Gauss's Law to a system of charges or charged region to determine characteristics of the electric field, flux, or charge contained in the system.
- State and use Gauss's Law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
- Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.
- Use Gauss's Law to calculate an unknown charge density or total charge on surface in terms of the electric field near the surface.
- Qualitatively describe electric fields around symmetrically (spherically, cylindrically, or planar) charged distributions.
- Describe the general features of an electric field due to symmetrically shaped charged distributions.
- Describe the general features of an unknown charge distribution, given other features of the system.
- Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.
- Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.
- Describe an electric field as a function of distance for the different types of symmetrical charge distributions.
- Derive expressions for the electric potential of a charge distribution using integration and the principle of superposition.
- Describe electric potential as a function of distance for the different types of symmetrical charge distributions.

- Identify regions of higher and lower electric potential by using a qualitative (or quantitative) argument to apply to the charged region of space.

Sample Performance Tasks

- Describe the physical meaning (includes identifying features) of a representation. (HS-PS2-6)
- Apply an appropriate law, definition, or mathematical relationship to solve a problem. (HS-PS2-1) (HS-PS2-4) (HS-ETS1-2)
- Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)
- Select and plot appropriate data. (HS-PS2-5)
- Create appropriate diagrams to represent physical situations. (HS-PS2-6)
- Identify and describe patterns and trends in data or a graph. (HS-PS3-2),(HS-PS3-5)
- Demonstrate consistency between different graphical representations of the same physical situation.(HS-PS2-5)
- Describe the relationship between different types of representations of the same physical situation.(HS-PS2-6)
- Determine the relationship between variables within an equation when an existing variable changes.(HS-PS2-2),(HS-PS2-4)
- Determine the relationship between variables within an equation when a new variable is introduced.(HS-PS2-2),(HS-PS2-4)
- Select an appropriate law, definition, mathematical relationship, or model to describe a physical situation. (HS-PS2-1) (HS-PS2-4)
- Support a claim with evidence from physical representations. (HS-PS2-6)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- The general definition of potential difference that can be used in most cases is:

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$$

or in the different form:

$$E_x = -\frac{dV}{dx}$$

- The general definition of electric flux is:

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

  - The definition for the total flux through a geometric closed surface is defined by the "surface integral" defined as:

$$\varphi_{closed} = \vec{E} \cdot d\vec{A}$$

  - The sign of the flux is given by the dot product between the electric field vector and the area vector.
  - The area vector is defined to be perpendicular to the plane of the surface and directed outward from a closed surface.
- Gauss's Law can be defined in a qualitative way as the total flux through a closed Gaussian surface being proportional to the charge enclosed by the Gaussian surface. The flux is also independent of the size of the Gaussian shape.
- Gauss's Law in integral form is:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

- In general, if a function of known charge density is given, the total charge can be determined using calculus, such as:

$$Q_{enc} = \int \rho(r) dV$$

The above is the general case for a volume-charge distribution.

- Gauss's Law can help in describing features of electric fields of charged systems at the surface, inside the surface, or at some distance away from the surface of charged objects.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} = \Phi_E$$

- Gauss's law can be useful in determining the charge distribution that created an electric field, especially if the distribution is spherically, cylindrically, or planarly symmetric.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} = \Phi_E$$

- The electric field of any charge distribution can be determined using the principle of superposition, symmetry, and the definition of electric field due to a differential charge dq. One step in the solution is shown to be:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

If this is applied appropriately and evaluated over the appropriate limits, the electric fields of the stated charge distributions can be determined as a function of position.
The following charge distributions can be explored using this method:
    - An infinitely long, uniformly charged wire or cylinder determine field at distances along perpendicular bisector
    - A thin ring of charge (along the axis of the ring)
    - A semicircular or part of a semicircular arc
    - A field due to a finite wire or line charge at a distance that is collinear with the line charge
- The general characteristics of electric fields can be proven from the calculus definitions (or Gauss's Law) and/or the principle of superposition.
The following electric fields can be explored:
    - Electric fields with planar symmetry, infinite sheets of charge, combinations of infinite sheets of charge, or oppositely charged plates
    - Linearly charged wires or charge distributions
    - Spherically symmetric charge distributions on spheres or spherical shells of charge
- Other distributions of charge that can be deduced using Gauss's Law or the principle of superposition.
- The integral definition of the electric potential due to continuous charge distributions is defined as: $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$
If this is applied appropriately and evaluated over the appropriate limits of integration, the potential due to the charge distribution can be determined as a function of position.
The following charge distributions can be explored using this method:
    - A uniformly charged wire
    - A thin ring of charge (along the axis of the ring)
    - A semicircular arc or part of a semicircular arc
    - A uniformly charged disk

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources
Core Text: Fundamentals of Physics 10th Edition, Halliday & Resnick, ISBN 978-1-118-23072-5 (Extended Edition) **Suggested Resources:** https://www.nextgenscience.org/sites/default/files/HS%20PS%20DCI%20combined%206.13.13.pdf https://apcentral.collegeboard.org/courses/ap-physics-c-mechanics?course=ap-physics-c-mechanics https://apcentral.collegeboard.org/courses/ap-physics-c-electricity-and-magnetism?course=ap-physics-c-electricity-and-magnetism

Unit Title: Unit 4 Capacitors and Electric Circuits	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11/12	
Summary and Rationale	
Previously, students investigated why all objects have an electric charge. In Unit 4, students will examine how that charge can move through an object. Conductors, capacitors, and dielectrics are presented to demonstrate that a charge's movement is dependent on an object's material. In electronics, each of these are important based on the type of movement or desired object behavior. Additionally, this unit examines how the behavior of these elements is impacted by electric fields. Students should be provided with opportunities (laboratory investigations or activities) to describe and examine the function of each of these elements, along with capacitors. Knowledge of conductors, capacitors, and dielectrics will prepare students for understanding how electric circuits work in Unit 4 and how they behave when one or more electrical element is altered or modified. Whether or not they're aware, students interact with electric circuits regularly through charging their phones, powering up their laptops, or simply switching on a light. Unit 4 serves to illuminate how, and why, our various appliances function by exploring the nature and importance of electric currents, circuits, and resistance. Through activities and lab investigations, students will have opportunities to relate knowledge across the course by using the electrical components they will learn about in Unit 4 and will come to discover in this unit to create, modify, and analyze circuits. Students will also analyze the relationships that exist between current, resistance, and power, in addition to exploring and applying Ohm's Law and Kirchhoff's Rules.	
Recommended Pacing	
17 days	
NGSS Standards/Performance Expectations	
HS-PS2-1	Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. Clarification Statement: Examples of data could include tables or graphs of position or velocity as a function of time for objects subject to a net unbalanced force, such as a falling object, an object rolling down a ramp, or a moving object being pulled by a constant force.
HS-PS3-1	Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known. [Clarification Statement: Emphasis is on explaining the meaning of mathematical expressions used in the model.] [Assessment Boundary: Assessment is limited to basic algebraic expressions or computations; to systems of two or three components; and to thermal energy, kinetic energy, and/or the energies in gravitational, magnetic, or electric fields.]
HS-PS3-2	Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as a combination of energy associated with the motions of particles (objects) and energy associated with the relative position of particles (objects). [Clarification Statement: Examples of phenomena at the macroscopic scale could include

	the conversion of kinetic energy to thermal energy, the energy stored due to position of an object above the earth, and the energy stored between two electrically-charged plates. Examples of models could include diagrams, drawings, descriptions, and computer simulations.]
HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.

Interdisciplinary Connections

Integration of Technology

Instructional Focus

Unit Enduring Understandings

- Excess charge on an insulated conductor will spread out on the entire conductor until there is no more movement of the charge.
- Excess charge on an insulated sphere or spherical shell will spread out on the entire surface of the sphere until there is no more movement of the charge because the surface is an equipotential.
- There are electrical devices that store and transfer electrostatic potential energy.
- An insulator has different properties (than a conductor) when placed in an electric field.
- The rate of charge flow through a conductor depends on the physical characteristics of the conductor.
- There are electrical devices that convert electrical potential energy into other forms of energy.
- Total energy and charge are conserved in a circuit containing resistors and a source of energy.
- Total energy and charge are conserved in a circuit that includes resistors, capacitors, and a source of energy.

Learning Objectives

Students will be able to:

- Calculate the electric potential on the surfaces of two charged conducting spheres when connected by a conducting wire.
- Apply the general definition of capacitance to a capacitor attached to a charging source.
- Calculate unknown quantities such as charge, potential difference, or capacitance for a physical system with a charged capacitor.
- Use the relationship for stored electrical potential energy for a capacitor.
- Calculate quantities such as charge, potential difference, capacitance, and potential energy of a physical system with a charged capacitor.
- Explain how a charged capacitor, which has stored energy, may transfer that energy into other forms of energy.
- Derive an expression for a parallel-plate capacitor in terms of the geometry of the capacitor and fundamental constants.
- Describe the properties of a parallel-plate capacitor in terms of the electric field between the plates, the potential difference between the plates, the charge on the plates, and distance of separation between the plates.
- Calculate physical quantities such as charge, potential difference, electric field, surface area, and distance of separation for a physical system that contains a charged parallel-plate capacitor.
- Explain how a change in the geometry of a capacitor will affect the capacitance value.
- Apply the relationship between the electric field between the capacitor plates and the surface-charge density on the plates.
- Derive expressions for the energy stored in a parallel-plate capacitor or the energy per volume of the capacitor.

- Describe the consequences to the physical system of a charged capacitor when a conduction slab is inserted between the plates or when the conducting plates are moved closer or farther apart.
- Calculate unknown quantities such as charge, potential difference, charge density, electric field, and stored energy when a conducting slab is placed in between the plates of a charged capacitor or when the plates of a charged capacitor are moved closer or farther apart.
- Derive expressions for a cylindrical capacitor or a spherical capacitor in terms of the geometry of the capacitor and fundamental constants.
- Use the definition of the capacitor to describe changes in the capacitance value when a dielectric is inserted between the plates.
- Calculate changes in energy, charge, or potential difference when a dielectric is inserted into an isolated charge capacitor.
- Calculate the terminal voltage and the internal resistance of a battery of specified EMF and known current through the battery.
- Calculate a single unknown current, potential difference, or resistance in a multi-loop circuit using Kirchhoff's Rules.
- Set up simultaneous equations to calculate at least two unknowns(currents or resistance values) in a multi-loop circuit.
- Explain why Kirchhoff's Rules are valid in terms of energy conservation and charge conservation around a circuit loop.
- Calculate the equivalent capacitance for capacitors arranged in series or parallel, or a combination of both, in steady-state situations.
- Calculate the potential difference across specified capacitors arranged in a series in a circuit.
- Calculate the stored charge in a system of capacitors and on individual capacitors arranged in series or in parallel.
- Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
- Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
- In transient circuit conditions (i.e., RC circuits), calculate the time constant of a circuit containing resistors and capacitors arranged in series.
- Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
- Recognize the model of charging or discharging a capacitor in an RC circuit, and apply the model to a new RC circuit.
- Describe stored charge or potential difference across a capacitor or current, or potential difference of a resistor in a transient RC circuit.
- Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.

Sample Performance Tasks

- Describe the physical meaning (includes identifying features) of a representation. (HS-PS2-6)
- Describe the effects of modifying conditions or features of a representation of a physical situation. (HS-PS2-2),(HS-PS2-4)
- Select an appropriate law, definition, mathematical relationship, or model to describe a physical situation. (HS-PS2-1) (HS-PS2-4)
- Support a claim with evidence from physical representations. (HS-PS2-6)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)
- Describe the physical meaning (includes identifying features) of a representation. (HS-PS2-2),(HS-PS2-4)
- Identify a testable scientific question or problem. (HS-PS3-2),(HS-PS3-5)
- Make a claim or predict the results of an experiment. (HS-PS3-4)
- Identify or describe potential sources of experimental error. (HS-PS3-3)
- Determine the relationship between variables within an equation when an existing variable changes. (HS-PS2-2),(HS-PS2-4)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)
- Sketch a graph that shows a functional relationship between two quantities. (HS-PS2-2),(HS-PS2-4)
- Create appropriate diagrams to represent physical situations. (HS-PS2-6)
- Determine the relationship between variables within an equation when an existing variable changes. (HS-PS2-2),(HS-PS2-4)
- Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)
- Support a claim with evidence from experimental data. (HS-PS2-6)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- The general definition of capacitance is given by the following relationship:
$$C = \frac{Q}{\Delta V}$$
- The energy stored in a capacitor is determined by the following relationship:
$$U_C = \frac{1}{2}C(\Delta V)^2$$
  (or an equivalent expression)
- The conservation of charge and energy can be applied to a closed physical system containing charge, capacitors, and a source of potential difference.
- The general definition of capacitance can be used in conjunction with the properties of the electric field of two large oppositely charged plates to determine the general definition for the parallel-plate capacitor in terms of the geometry of that capacitor. The relationship is:
$$C = \frac{\varepsilon_0 A}{d}$$
  where A is the surface area of a plate and d is the distance of separation between the plates. The plates in a capacitor can be considered to have a very large surface area compared with the distance of separation between the plates. This condition makes this an ideal capacitor with a constant electric field between the plates.
- The electric field of oppositely charged plates can be determined by applying Gauss's Law or by applying the principle of superposition. The electric field between the two plates of a parallel-plate capacitor has the following properties:
  - The electric field is constant in magnitude and is independent of the geometry of the capacitor.
  - The electric field is proportional to the surface-charge density of the charge on one plate.

- The energy of the parallel-plate capacitor can be expressed in terms of the fundamental properties of the capacitor (i.e., area, distance of separation), fundamental properties of the charged system (i.e., charge density), and fundamental constants.
- The charged-capacitor system will have different conserved quantities depending on the initial conditions or conditions of the capacitor. If the capacitor remains attached to a source of a potential difference, then the charge in the system can change in accordance with the changes to the system. If the capacitor is isolated and unattached to a potential source, then the charge in the capacitor system remains constant and other physical quantities can change in response to changes in the physical system.
- Using the definition of capacitance and the properties of electrostatics of charged cylinders or spheres, the capacitance of a cylindrical or spherical capacitor can also be determined in terms of its geometrical properties and fundamental constants.
- The properties of capacitance still hold for all types of capacitors (spherical or cylindrical).
- An insulator's molecules will polarize to various degrees (slightly polarize or largely polarize). This effect is determined by a physical constant called the "dielectric constant." The dielectric constant has values between 1 and larger numbers.
- The dielectric will become partially polarized and create an electric field inside of the dielectric material. The net electric field between the plates of the capacitor is the resultant of the two fields—the fields between the plates and the induced field in the dielectric medium. This field is always a reduction in the field between the plate and therefore a reduction in the potential difference between the plates.
- The capacitance of a parallel-plate capacitor with a dielectric material inserted between the plates can be calculated as follows:

  $$C = \frac{\kappa \varepsilon_0 A}{d}$$

  where the constant κ is the dielectric constant of the material.
- The initial condition of the capacitor system can determine which relationship to use when attempting to calculate unknown quantities in a capacitor system.
- In a nonideal battery, an internal resistance will exist within the battery. This resistance will add in series to the total external circuit resistance and reduce the operating current in the circuit.
- Kirchhoff's Rules allow for the determination of currents and potential differences in complex multi-loop circuits that cannot be reduced using conventional (series/parallel rules) methods.
  - According to Kirchhoff's current rule ,the current into a junction or node must be equal to the current out of that junction or node. This is a consequence of charge conservation.
  - According to Kirchhoff's loop rule, the sum of the potential differences around a closed loop must be equal to zero. This is a consequence of the conservation of energy in a circuit loop.
- The equivalent capacitance of capacitors arranged in series can be determined by the following relationship:

  $$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

  - The equivalent capacitance of capacitors arranged in parallel can be determined by the following relationship:

  $$C_{eq} = \sum_i C_i$$

  - The system of capacitors will behave as if the one equivalent capacitance were connected to the voltage source.

  - For capacitors arranged in parallel, the total charge stored in the system is equivalent to the sum of the individual stored charges on each capacitor.
  - For capacitors arranged in series, the total stored charge in the system is Q_T, and each individual capacitor also has a charge value of Q_T.
- When a circuit containing resistors and capacitors reaches a steady-state condition, the potential difference across the capacitor can be determined using Kirchhoff's Rules.
- Under transient conditions for *t* = 0 to *t* = steady-state conditions, the time constant in an RC circuit is equal to the product of equivalent resistance and the equivalent capacitance.
- The changes in the electrical characteristics of a capacitor or resistor in an RC circuit can be described by fundamental differential equations that can be integrated over the transient time interval.
  - The general model for the charging or discharging of a capacitor in an RC circuit contains a factor of $e^{-\frac{t}{RC}}$.
- The time constant ($\tau = RC$) is a significant feature on the sketches for transient behavior in an RC circuit.
  - These particular sketches will always have the exponential decay factor and will either have an asymptote of zero or an asymptote that signifies some physical final state of the system (e.g., final stored charge).
  - The initial conditions of the circuit will be represented on the sketch by the vertical intercept of the graph (e.g., initial current).
  - The capacitor in a circuit behaves as a "bare wire" with zero resistance at a time immediately after *t* = 0 seconds.
  - The capacitor in a circuit behaves as an "open circuit" or having an infinite resistance in a condition of time much greater than the time constant of the circuit.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Fundamentals of Physics 10th Edition, Halliday & Resnick, ISBN 978-1-118-23072-5 (Extended Edition)

Suggested Resources:

https://www.nextgenscience.org/sites/default/files/HS%20PS%20DCI%20combined%206.13.13.pdf

https://apcentral.collegeboard.org/courses/ap-physics-c-mechanics?course=ap-physics-c-mechanics

https://apcentral.collegeboard.org/courses/ap-physics-c-electricity-and-magnetism?course=ap-physics-c-electricity-and-magnetism

Unit Title: Unit 5 Magnetism	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11-12	
Summary and Rationale	
In previous units, students discovered that the electric field allows charged objects to interact without contact. Unit 5 introduces students to magnetism and how magnetic fields are generated, behave, and relate to electricity. Students will learn how magnetic fields impact motion and interact with other magnetic fields. Laboratory investigations and/or activities should be provided for students to apply both the Biot–Savart Law (using calculations to determine the strength of a magnetic field) and Ampère's Law (deriving mathematical relationships which relate the magnitude of the magnetic field to current). This knowledge from previous units helps students to make connections between electric fields and magnetic fields as well as between Gauss's Law and Ampère's Law.	
Recommended Pacing	
18 days	
NGSS Standards/Performance Expectations	
HS-PS2-5	Plan and conduct an investigation to provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.
HS-PS3-5	Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction. (Clarification Statement: Examples of models could include drawings, diagrams, and texts, such as drawings of what happens when two charges of opposite polarity are near each other.)
HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
Interdisciplinary Connections	
Integration of Technology	
Instructional Focus	
Unit Enduring Understandings	
● Current-carrying conductors create magnetic fields that allow them to interact at a distance with other magnetic fields. ● There are laws that use symmetry and calculus to derive mathematical relationships that are applied to physical systems containing moving charge.	
Learning Objectives	
Students will be able to: ● Calculate the magnitude and direction of a magnetic field produced at a point near a long, straight, current-carrying wire. ● Apply the right-hand rule for magnetic field of a straight wire (or correctly use the Biot–Savart Law) to deduce the direction of a magnetic field near a long, straight, current-carrying wire. ● Describe the direction of a magnetic-field vector at various points near multiple long, straight, current-carrying wires.	

- Calculate the magnitude of a magnetic field at various points near multiple long, straight, current-carrying wires.
- Calculate an unknown current value or position value, given a specified magnetic field at a point due to multiple long, straight, current-carrying wires.
- Calculate the force of attraction or repulsion between two long, straight, current-carrying wires.
- Describe the consequence (attract or repel) when two long, straight, current-carrying wires have known current directions.
- Describe the direction of the contribution to the magnetic field made by a short (differential) length of straight segment of a current-carrying conductor.
- Calculate the magnitude of the contribution to the magnetic field due to a short (differential) length of straight segment of a current-carrying conductor.
- Derive the expression for the magnitude of magnetic field on the axis of a circular loop of current or a segment of a circular loop.
- Explain how the Biot–Savart Law can be used to determine the field of a long, straight, current-carrying wire at perpendicular distances close to the wire.
- Explain Ampere's Law and justify the use of the appropriate Amperian loop for current-carrying conductors of different shapes such as straight wires, closed circular loops, conductive slabs, or solenoids.
- Derive the magnitude of the magnetic field for certain current-carrying conductors using Ampere's Law and symmetry arguments.
- Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampere's Law.
- Describe the conclusions that can be made about the magnetic field at a particular point in space if the line integral in Ampere's Law is equivalent to zero.
- Describe the relationship of the magnetic field as a function of distance for various configurations of current-carrying cylindrical conductors with either a single current or multiple currents, at points inside and outside of the conductors.
- Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.
- Calculate the magnitude of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.

Sample Performance Tasks

- Make a claim or predict the results of an experiment. (HS-PS3-4)
- Create appropriate diagrams to represent physical situations. (HS-PS2-6)
- Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)
- Support a claim with evidence from physical representations. (HS-PS2-6)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)
- Identify appropriate experimental procedures (which may include a sketch of a lab setup). (HS-PS3-4)
- Make observations or collect data from representations of laboratory setups or results. (HS-PS2-5)
- Explain modifications to an experimental procedure that will alter results. (HS-PS3-3)
- Select and plot appropriate data. (HS-PS2-2),(HS-PS2-4)
- Apply an appropriate law, definition, or mathematical relationship to solve a problem. (HS-PS2-1) (HS-PS2-4) (HS-ETS1-2)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)
- Explain the connection between experimental results and larger physical principles, laws, or theories. (HS-PS2-1) (HS-PS2-4)

- Explain how potential sources of experimental error may affect results and/or conclusions. (HS-PS2-6)
- Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units. (HS-PS2-1)
- Sketch a graph that shows a functional relationship between two quantities. (HS-PS2-2),(HS-PS2-4)
- Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. (HS-PS2-1)
- Determine or estimate the change in a quantity using a mathematical relationship. (HS-PS2-2),(HS-PS2-4)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- It can be shown or experimentally verified that the magnetic field of a long, straight, current-carrying conductor is:

$$B = \frac{\mu_0 I}{2\pi r}$$

  - The magnitude of the field is proportional to the inverse distance from the wire.
  - The magnetic-field vector is always mutually perpendicular to the position vector and the direction of the conventional current. The result of this is a magnetic field line that is in a circular path around the wire in a sense (clockwise or counterclockwise) determined by the appropriate right-hand rule.
  - The magnetic field inside a solenoid can be determined using:

$$B = \mu_0 n I$$

- The principle of superposition can be used to determine the net magnetic field at a point due to multiple long, straight, current-carrying wires.
- The field of a long, straight wire can be used as the external field in the definition of magnetic force acting on a segment of current carrying wire.
  - The direction of the force can be determined from the cross-product definition or from the appropriate right-hand rule.
- The Biot–Savart Law is the fundamental law of magnetism that defines the magnitude and direction of a magnetic field due to moving charges or current-carrying conductors. The law in differential form is:

$$dB = \frac{\mu_0}{4\pi} \frac{I(d\ell \times \hat{r})}{r^2}$$

- The Biot–Savart Law can be used to derive the magnitude and directions of magnetic fields of symmetric current-carrying conductors (e.g., circular loops), long, straight conductors, or segments of loops.
- Ampère's Law is a fundamental law of magnetism that relates the magnitude of the magnetic field to the current enclosed by a closed imaginary path called an Amperian loop. The law in integral form is:

$$\oint B \cdot d\ell = \mu_0 I$$

  where I in this case is the enclosed current by the Amperian loop.
  - Ampère's Law for magnetism is analogous to Gauss's Law for electrostatics and is a fundamental law that allows for an easier approach to determining some magnetic fields of certain symmetries or shapes of current-carrying conductors. The law is always true but not always useful.
  - The law can only be applied when the symmetry of the magnetic field can be exploited. Circular loops; long, straight wires; conductive slabs with current density; solenoids; and other cylindrical conductors containing current are the types of shapes for which Ampère's Law can be useful.

- Ampère’s Law can be used to determine magnetic-field relationships at different locations in cylindrical current-carrying conductors.
- The principle of superposition can be used to determine the net magnetic field at a point in space due to various combinations of current-carrying conductors, loops, segments, or cylindrical conductors. Ampère’s Law can be used to determine individual field magnitudes. The principle of superposition can be used to add those individual fields.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

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https://apcentral.collegeboard.org/courses/ap-physics-c-mechanics?course=ap-physics-c-mechanics

https://apcentral.collegeboard.org/courses/ap-physics-c-electricity-and-magnetism?course=ap-physics-c-electricity-and-magnetism

Unit Title: Unit 6 Electromagnetism	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11-12	
Summary and Rationale	
Throughout the course, students explored the vital roles electricity and magnetism play in our daily lives. Unit 6 examines electromagnetism through the concept of electromagnetic induction and the application of Maxwell's equations. Through activities and detailed laboratory investigations, students will study, apply, and analyze the concept of induction, as well as investigate the relationship between Faraday's Law and Lenz's Law. Additionally, students are expected to call upon their knowledge obtained in earlier units—particularly that of charges, currents, and electric and magnetic fields—to better understand Maxwell's equations and to be able to mathematically demonstrate, as well as reason with, how these fields are generated.	
Recommended Pacing	
16 days	
NGSS Standards/Performance Expectations	
HS-PS2-5	Plan and conduct an investigation to provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.
HS-PS3-5	Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction. (Clarification Statement: Examples of models could include drawings, diagrams, and texts, such as drawings of what happens when two charges of opposite polarity are near each other.)
HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.
Interdisciplinary Connections	
Integration of Technology	
Instructional Focus	
Unit Enduring Understandings	
● There are laws that use symmetry and calculus to derive mathematical relationships that are applied to physical systems containing a magnetic field. ● A changing magnetic field over time can induce current in conductors. ● Induced forces (arising from magnetic interactions) that are exerted on objects can change the kinetic energy of an object. ● In a closed circuit containing inductors and resistors, energy and charge are conserved. ● Electric and magnetic fields that change over time can mutually induce other electric and magnetic fields.	

Learning Objectives
Students will be able to: ● Calculate the magnetic flux through a loop of regular shape with an arbitrary orientation in relation to the magnetic-field direction. ● Calculate the magnetic flux of the field due to a current-carrying, long, straight wire through a rectangular-shaped area that is in the plane of the wire and oriented perpendicularly to the field. ● Calculate the magnetic flux of a non-uniform magnetic field that may have a magnitude that varies over one coordinate through a specified rectangular loop that is oriented perpendicularly to the field. ● Describe which physical situations with a changing magnetic field and a conductive loop will create an induced current in the loop. ● Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field. ● Describe the induced current magnitudes and directions for a conductive loop moving through a specified region of space containing a uniform magnetic field. ● Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate. ● Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when a physical quantity related to magnetic field or area is changing with a specified non-linear function of time. ● Derive expressions for the induced EMF (or current) through a closed conductive loop with a time-varying magnetic field directed either perpendicularly through the loop or at some angle oriented relative to the magnetic-field direction. ● Describe the relative magnitude and direction of induced currents in a conductive loop with a time-varying magnetic field. ● Determine if a net force or net torque exists on a conductive loop in a region of changing magnetic field. ● Justify if a conductive loop will change its speed as it moves through different regions of a uniform magnetic field. ● Calculate an expression for the net force on a conductive bar as it is moved through a magnetic field. ● Write a differential equation and calculate the terminal velocity for the motion of a conductive bar (in a closed electrical loop) falling through a magnetic field or moving through a field due to other physical mechanisms. ● Describe the mechanical consequences of changing an electrical property (such as resistance) or a mechanical property (such as length/area) of a conductive loop as it moves through a uniform magnetic field. ● Derive an expression for the mechanical power delivered to a conductive loop as it moves through a magnetic field in terms of the electrical characteristics of the conductive loop. ● Derive the expression for the inductance of a long solenoid. ● Calculate the magnitude and the sense of the EMF in an inductor through which a changing current is specified. ● Calculate the rate of change of current in an inductor with a transient current. ● Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors. ● Calculate the maximum current in a circuit that contains only a charged capacitor and an inductor. ● Derive a differential equation for the current as a function of time in a simple LR series circuit. ● Derive a solution to the differential equation for the current through the circuit as a function of time in the cases involving the simple LR series circuit.

- Describe currents or potential differences with respect to time across resistors or inductors in a simple circuit containing resistors and an inductor, either in series or a parallel arrangement.
- Explain how a changing magnetic field can induce an electric field.
- Associate the appropriate Maxwell's equation with the appropriate physical consequence in a physical system containing a magnetic or electric field.

Sample Performance Tasks

- Select relevant features of a representation to answer a question or solve a problem. (HS-PS2-1) (HS-PS2-4) (HS-ETS1-2)
- Describe the effects of modifying conditions or features of a representation of a physical situation. (HS-PS2-2),(HS-PS2-4)
- Assess the reasonableness of results or solutions. (HS-ETS1-3)
- Make a scientific claim. (HS-PS2-5)
- Explain the connection between experimental results and larger physical principles, laws, or theories.(HS-PS2-1) (HS-PS2-4)
- Select an appropriate law, definition, mathematical relationship, or model to describe a physical situation. (HS-PS2-1) (HS-PS2-4)
- Apply an appropriate law, definition, or mathematical relationship to solve a problem. (HS-PS2-1) (HS-PS2-4) (HS-ETS1-2)
- Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)
- Provide reasoning to justify a claim using physical principles or laws. (HS-PS2-1) (HS-PS2-4)
- Linearize data and/or determine a best fit line or curve. (HS-PS2-2),(HS-PS2-4)
- Explain how the data or graph illustrates a physics principle, process, concept, or theory. (HS-PS2-1)
- Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. (HS-PS2-1)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- Magnetic flux is the scalar product of the magnetic-field vector and the area vector over the entire area contained by the loop. The definition of magnetic flux is:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- Induced currents arise in a conductive loop (or long wire) when there is a change in magnetic flux occurring through the loop. This change is defined by Faraday's Law:

$$\varepsilon_i = -N\frac{d\Phi_B}{dt}$$

where ei is the induced EMF and N is the number of turns. (In a coil or solenoid, the N refers to the number of turns of coil or conductive loops in the solenoid.)
  - The negative sign in the expression embodies Lenz's Law and is an important part of the relationship.
  - Lenz's Law is the relationship that allows the direction of the induced current to be determined. The law states that any induced EMF and current induced in a conductive loop will create an induced current and induced magnetic field to oppose the direction change in external flux.
  - Lenz's Law is essentially a law relating to conservation of energy in a system and has mechanical consequences.
- When an induced current is created in a conductive loop, the current will interact with the already-present magnetic field, creating induced forces acting on the loop. The magnitude and directions of these induced forces can be calculated using the definition of force on a current-carrying wire.

- Newton's second law can be applied to a moving conductor as it experiences a flux change.
  - The force on the conductor is proportional to the velocity of the conductor.
  - A different equation of velocity can be written for these physical situations.
  - This will lead to an exponential relationship with the changing velocity of the conductor.
  - Using calculus, the expressions for velocity, induced force, and power can all be expressed with these exponential relationships.
- By applying Faraday's Law to an inductive electrical device, a variation on the law can be determined to relate the definition of inductance to the properties of the inductor:

$$\varepsilon_L = -L\frac{di}{dt}$$

where L is defined as the inductance of the electrical device.

  - The very nature of the inductor is to oppose the change in current occuring in the inductor.
- The stored energy in an inductor is defined by:

$$U_B = \frac{1}{2}Li^2$$

- The electrical characteristics of an inductor in a circuit are the following:
  - At the initial condition of closing or opening a switch with an inductor in a circuit, the induced voltage will be equal in magnitude and opposite in direction of the applied voltage across the branch containing the inductor.
  - In a steady-state condition, the ideal inductor has a resistance of zero and therefore will behave as a bare wire in a circuit.
  - In circuits containing only a charged capacitor and an inductor, the maximum current through the inductor can be determined by applying conservation of energy within the circuit and the two circuit elements that can store energy.
- Kirchhoff's Rules can be applied to a series LR circuit. The result of applying Kirchhoff's rules in this case will be a differential equation in current for the loop.
  - The solution of this equation will yield the fundamental models for the LR circuit (in turning on the circuit and turning off the circuit).
- Using Kirchhoff's Rules and the general model for an LR circuit, general current characteristics can be determined in an LR circuit in a series or parallel arrangement.
- Maxwell's Laws completely describe the fundamental relationships of magnetic and electric fields in steady-state conditions, as well as in situations in which the fields change in time.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Fundamentals of Physics 10th Edition, Halliday & Resnick, ISBN 978-1-118-23072-5 (Extended Edition)

Suggested Resources:

https://www.nextgenscience.org/sites/default/files/HS%20PS%20DCI%20combined%206.13.13.pdf

https://apcentral.collegeboard.org/courses/ap-physics-c-mechanics?course=ap-physics-c-mechanics

https://apcentral.collegeboard.org/courses/ap-physics-c-electricity-and-magnetism?course=ap-physics-c-electricity-and-magnetism

Unit Title: Unit 7- Mechanics Revisited	
Content Area: Science	
Course & Grade Level: Advanced Topics in Physics Honors, 11/12	
Summary and Rationale	
In this unit, students will revisit previous principles learned in mechanics. Students have learned many of these topics at an introductory level in the honors course, but in this unit, those skills will be honed with the addition of calculus. Forces, orbital motion, air drag, energy, and simple harmonic motion will all be reviewed and discussed as calculus is brought in to help bridge the mathematical gaps that were missing from the previous year. While this unit does cover a range of topics, it is done quickly as a strong base for this unit has already been established previously. During their study, students will gain a more in-depth understanding of motion, making them better equipped to apply their knowledge of forces and motion to real world surroundings.	
Recommended Pacing	
15 days	
NGSS Standards/Performance Expectations	
HS-PS2-1	Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. Clarification Statement: Examples of data could include tables or graphs of position or velocity as a function of time for objects subject to a net unbalanced force, such as a falling object, an object rolling down a ramp, or a moving object being pulled by a constant force.
HS-PS3-1	Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known. [Clarification Statement: Emphasis is on explaining the meaning of mathematical expressions used in the model.] [Assessment Boundary: Assessment is limited to basic algebraic expressions or computations; to systems of two or three components; and to thermal energy, kinetic energy, and/or the energies in gravitational, magnetic, or electric fields.]
HS-PS3-2	Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as a combination of energy associated with the motions of particles (objects) and energy associated with the relative position of particles (objects). [Clarification Statement: Examples of phenomena at the macroscopic scale could include the conversion of kinetic energy to thermal energy, the energy stored due to position of an object above the earth, and the energy stored between two electrically-charged plates. Examples of models could include diagrams, drawings, descriptions, and computer simulations.]
HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.

Interdisciplinary Connections

Integration of Technology

Instructional Focus

Unit Enduring Understandings

- There are relationships among the vector quantities of position, velocity, and acceleration for the motion of a particle along a straight line.
- There are multiple simultaneous relationships among the quantities of position, velocity, and acceleration for the motion of a particle moving in more than one dimension with or without net forces.
- A net force will change the translational motion of an object.
- Objects of large mass will cause gravitational fields that create an interaction at a distance with other objects with mass.
- Angular momentum and total mechanical energy will not change for a satellite in an orbit.
- There are certain types of forces that cause objects to repeat their motions with a regular pattern.

Learning Objective

Students will be able to:

- Derive an expression for the motion of an object freely falling with a resistive drag force (or moving horizontally subject to a resistive horizontal force).
- Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).
- Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.
- Derive a differential equation for an object in motion subject to a specified resistive force.
- Derive an expression for a time-dependent velocity function for an object moving under the influence of a given resistive force (with given initial conditions).
- Derive expressions for the acceleration or position of an object moving under the influence of a given resistive force.
- Calculate quantitative properties (such as period, speed, radius of orbit) of a satellite in circular orbit around a planetary object.
- Calculate the gravitational potential energy and the kinetic energy of a satellite/Earth system in which the satellite is in circular orbit around the earth.
- Derive the relationship of total mechanical energy of a satellite/earth system as a function of radial position.
- Derive an expression for the escape speed of a satellite using energy principles.
- Describe the motion of a satellite launched straight up (or propelled toward the planet) from the planet's surface, using energy principles.
- Calculate positions, speeds, or energies of a satellite launched straight up from the planet's surface, or a satellite that is projected straight toward the planet's surface, using energy principles.
- Calculate the orbital distances and velocities of a satellite in elliptical orbit using the conservation of angular momentum.
- Calculate the speeds of a satellite in elliptical orbit at the two extremes of the elliptical orbit (perihelion and aphelion).
- Describe the general behavior of a spring-mass system in SHM in qualitative terms.
- Describe the relationship between the phase angle and amplitude in an SHM system.
- Describe the displacement in relation to time for a mass-spring system in SHM.
- Identify the period, frequency, and amplitude of the SHM in a mass-spring system from the features of a plot.

- Describe each of the three kinematic characteristics of a spring-mass system in SHM in relation to time (displacement, velocity, and acceleration). For a spring-mass system in SHM—
    - Describe the general features of the motion and
    - Identify the places on a graph where these values are zero or have maximum positive values or maximum negative values.
- Derive the expression for the period of oscillation for various physical systems oscillating in SHM.
- Calculate the mechanical energy of an oscillating system. Show that this energy is conserved in an ideal SHM spring-mass system.
- Describe the effects of changing the amplitude of a spring-mass system.
- Describe a linear relationship between the period of a system oscillating in SHM and physical constants of the system.

Sample Performance Tasks

- Describe the effects of modifying conditions or features of a representation of a physical situation. (HS-PS2-2),(HS-PS2-4)
- Make a claim or predict the results of an experiment. (HS-PS2-5)
- Explain modifications to an experimental procedure that will alter results. (HS-PS3-3)
- Linearize data and/or determine a best fit line or curve. (HS-PS2-2),(HS-PS2-4)
- Explain how the data or graph illustrates a physics principle, process, concept, or theory. (HS-PS2-1)
- Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. (HS-PS2-1)
- Explain how potential sources of experimental error may affect results and/or conclusions. (HS-PS2-6)
- Create appropriate diagrams to represent physical situations. (HS-PS2-6)
- Explain how the data or graph illustrates a physics principle, process, concept, or theory. (HS-PS2-1)
- Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. (HS-PS2-1)
- Sketch a graph that shows a functional relationship between two quantities. (HS-PS2-2),(HS-PS2-4)
- Determine or estimate the change in a quantity using a mathematical relationship. (HS-PS2-2),(HS-PS2-4)
- Calculate an unknown quantity with units from known quantities by selecting and following a logical computational pathway. (HS-PS2-2),(HS-PS2-4)

Evidence of Learning - Disciplinary Core Ideas

Essential Knowledge

- The standard "resistive force" in this course is defined as a velocity-dependent force in the opposite direction of velocity, for example:

$$F_r = -kv$$

or

$$|F_r| = kv^2$$

- The terminal velocity is defined as the maximum speed achieved by an object falling under the influence of a given drag force. The terminal condition is reached when the magnitude of the drag force is equal to the magnitude of the weight of the object.
- Because the resistive force is a function of velocity, applying Newton's second law correctly will lead to a differential equation for velocity. This is an example of that statement:

$$\frac{dv}{dt} = -\frac{k}{m}v$$

    - Using the method of separation of variables, the velocity can be determined from relationships by correctly integrating over the proper limits of integration.

- The acceleration or position can be determined using methods of calculus once a function for velocity is determined.
- In ideal situations, the energy in a planet/satellite system is a constant.
  - The gravitational potential energy of a planet/satellite system is defined to have a zero value when the satellite is at an infinite distance (very large planetary distance) away from the planet.
  - By definition, the "escape speed" is the minimum speed required to escape the gravitational field of the planet. This could occur at a minimum when the satellite reaches a nominal speed of approximately zero at some very large distance away from the planet.
- In ideal non-orbiting cases, a satellite's physical characteristics of motion can be determined using the conservation of energy.
- In all cases of orbiting satellites, the total angular momentum of the satellite is a constant.
  - The conservation of mechanical energy and the conservation of angular momentum can both be used to determine speeds at different positions in the elliptical orbit.
- The general relationship for SHM is given by the following relationship:

$$x = x_{max}\cos(\omega t + \varphi)$$

  φ is the phase angle and x_{max} is the amplitude of the oscillation. This expression can be simplified given initial conditions of the system.
- The period of SHM is related to the angular frequency by the following relationship:

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

- Using calculus and the position in relation to time relationship for an object in SHM, all three kinematic characteristics can be explored. Recognizing the positions or times where the trigonometric functions have extrema or zeroes can provide more detail in qualitatively describing the behavior of the motion.
- Using Newton's second law, the following characteristic differential equation of SHM can be derived:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The physical characteristics of the spring-mass system (or pendulum) can be determined from the differential relationship.
- All of the characteristics of motion in SHM can be determined by using the general relationship $x = x_{max}\cos(\omega t + \varphi)$ and calculus relationships.
- The period can be derived from the characteristic differential equation. The following types of SHM systems can be explored:
  - Mass oscillating on spring in vertical orientation
  - Mass oscillating on spring in horizontal orientation
  - Mass-spring system with springs in series or parallel
  - Simple pendulum
  - Physical pendulum
  - Torsional pendulum
- Potential energy can be calculated using the spring constant and the displacement from equilibrium of a mass-spring system:

$$U_s = \frac{1}{2}k(\Delta x)^2$$

  - Mechanical energy is always conserved in an ideal oscillating spring-mass system.
  - Maximum potential energy occurs at maximum displacement, where velocity is zero and kinetic energy is zero. This maximum potential energy is equivalent to the total mechanical energy of the system.
  - These energy relationships are true in the following three types of SHM systems:
    - Mass-spring in horizontal orientation

        - Mass-spring in vertical orientation
        - Simple pendulum
- Total energy of a spring-mass system is proportional to the square of the amplitude.

$$E_{total} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

    - The total energy is composed of the two contributing mechanical energies of the spring-mass system.

$$E_{total} = K + U_s$$

- The total mechanical energy of a system in SHM is conserved. The potential energy of the spring-mass system is:

$$U_s = \frac{1}{2}k(\Delta x)^2$$

  and the kinetic energy of the system is:

$$K = \frac{1}{2}mv^2$$

  The total energy in the system is defined above.
- Any physical system that creates a linear restoring force ($F_{spring} = -k\Delta x$) will exhibit the characteristics of SHM.
- The period of a system oscillating in SHM is:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

  (or its equivalent for a pendulum or physical pendulum) and this can be shown to be true experimentally from a plot of the appropriate data.

$$T_p = 2\pi\sqrt{\frac{l}{g}}$$

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